Solutions to Homework 6

1. a) One kg of solar material contains 0.7kg of Hydrogen. We find the number of Hydrogen atoms by dividing this mass by the mass of one single Hydrogen atom: $N_H = 0.7kg/1.7 \cdot 10^{-27}kg \simeq 4 \cdot 10^{26}$.
b) Since 4 Hydrogen atoms are needed in order to produce 1 He atom, then $1 \cdot 10^{26}$ He atoms are produced from the $4 \cdot 10^{26}$ H atoms in 1 kg of solar material.
c) The production of one He atom is associated with a mass deficit of $\Delta m \simeq 5 \cdot 10^{-29}kg$. The total mass that is converted into energy out of 1 kg of solar material will be: $\Delta m \cdot N_{He} = 5 \cdot 10^{-29}kg \cdot 10^{26} = 5 \cdot 10^{-3}kg$.
d) Using the equivalence between mass and energy $E = m \cdot c^2$, we can convert the above mass into energy: $E = 5 \cdot 10^{-3}kg \cdot (3 \cdot 10^8 m/s)^2 = 4.5 \cdot 10^{14}J$.
e) If one kg of solar material produces $4.5 \cdot 10^{14}J$, then in order to account for the Sun's energy production of $4 \cdot 10^{26}J/s$, $4 \cdot 10^{26}J/s/(4.5 \cdot 10^{14}J/kg) = 9 \cdot 10^{11}kg$ of material need to be burned per second.
f) In one year that would amount to $3.15 \cdot 10^7 s \cdot 9 \cdot 10^{11}kg/s \simeq 2.8 \cdot 10^{19}kg$. Note that we first transformed $1y = 3.15 \cdot 10^7 s$.
g) With the solar mass being about $2 \cdot 10^{30}kg$, we get a lifetime of $2 \cdot 10^{30}kg/(2.8 \cdot 10^{19}kg/y) = 70 \cdot 10^{10}y$, or 70 billion years. Which is too large.
h) If only 10% of the solar mass (the core) is used up to create energy through nuclear fusion, then the lifetime is 10% of the above found value, so only 7 billion years. This is pretty close to its actually known lifetime of 11 billion years. The Sun was dimmer in the past and the core may be somewhat larger as well, which is why the lifetime we found out is rather a lower limit.

2.) According to the inverse square law, the star 5 times farther is 25 times fainter, or the closer star is 25 times brighter. (The brightness is inversely proportional to the distance squared.)

3.) The primary chemical component of most stars is Hydrogen.

M2 stars are cool, and so their atoms have many more bound electrons, as well as molecules can exist with their own wealth of spectral lines.

TiO stops at roughly 3500K when its molecular bonds get broken because it is getting too hot.

4.) Sirius's distance is $d = 1/p = 1/0.4 = 2.5pc$. The distance in light years = distance in parsecs $\times 3.26 Ly/pc$, which is about 8Ly.

5.) From Pluto we would have a a larger baseline, precisely 40 times larger than from Earth, since Pluto is 40 times farther away from the Sun than the Earth. This changes the parallax formula to: $d = 40/p$ and $p = 40/d = 40/2.5 = 16 arcsec$. From Pluto Sirius would have a 40 times larger parallax. We could measure the distance to stars that are 40 times farther away than we can from Earth. The smallest parallax we can measure from Earth is about 0.01 arcsec, which translates into a distance of 100pc. The farthest stars that we could measure from Pluto would be 4000pc away, which is about 7.5 times less than the diameter of our galaxy. The drawback, however, would be that we would have to wait for a time equal to half of Pluto's orbital period in order to establish the baseline, and this is about 126 years. We would never find out during our lifetime how far the star is.

6.) The total mass of the binary system can be calculated from Kepler's 3rd law: $\frac{a^3}{P^2} = \frac{M_1+M_2}{M_\odot}$, where the units of the semi-major axis need to be AU and the units of the period need to be $years$. The mass is then given in units of the solar mass.
$M_1 + M_2 = \frac{20^3}{50^2} = 3.2 M_\odot$. (Note $M_\odot$ is the astronomical abbreviation for "solar mass". The symbol $\odot$ refers to the Sun.

7.) The star with the smaller wobble (Sirius A) is more massive, because among two objects subject to the same gravitational force (Newton's 3rd law), the more massive object will experience less of an acceleration.
Using the sum of their masses from the previous problem and substituting $M_1 = 2.2 M_2$ we get $3.2 M_2 = 3.2 M_\odot$, or $M_2 = 1 M_\odot$. Sirius B has one solar mass. $M_1 = (3.2-1) M_\odot = 2.2 M_\odot$. Sirius A has 2.2 solar masses.

8.) From Table A-4 the luminosity of Sirius A is $L_A = 26 L_\odot$, and the luminosity of Sirius B is $L_B = 2.4 \cdot 10^{-3} L_\odot$. Note that luminosities are expressed in units of the solar luminosity. The temperature can be read off the Hertzsprung-Russel diagram (Fig. 10-4). We can estimate $T_A = 10,000K \simeq 1.7 T_\odot$ and $T_B = 24,000K \simeq 4 T_\odot$. Both temperatures are expressed in units of the Sun's temperature ($T_\odot = 5800K$). Applying Stefan-Boltzmann's law for the total luminosity, $L = 4\pi\sigma \cdot R^2 \cdot T^4$ once for each of the stars and once for the Sun and dividing the expressions by each other, we obtain:
$\frac{L_A}{L_\odot} = \frac{4\pi\sigma \cdot R_A^2 \cdot T_A^4}{4\pi\sigma \cdot R_\odot^2 \cdot T_\odot^4} = \frac{R_A^2 \cdot T_A^4}{R_\odot^2 \cdot T_\odot^4} = (R_A/R_\odot)^2 \cdot (T_A/T_\odot)^4$. We solve for the ratio $R_A/R_\odot$, which means that we will get the radius expressed in units of the solar radius:
$R_A/R_\odot = \sqrt{(L_A/L_\odot)}/(T_A/T_\odot)^2 = \sqrt{26}/2.89 \simeq 1.76$ or $R_A = 1.76 \cdot R_\odot$.
Following the same procedure for Sirius B, we obtain:

$R_B/R_\odot = \sqrt{(L_B/L_\odot)}/(T_B/T_\odot)^2 = \sqrt{2.4 \cdot 10^{-3}}/16 \simeq 3 \cdot 10^{-3}$, or $R_B = 3 \cdot 10^{-3} \cdot R_\odot$, or expressed in Earth radii (one Earth radius being about 100 times smaller than the solar radius), $R_B = 0.3R_{Earth}$.

The ratio of luminosities is related to the difference of absolute magnitudes $M_B - M_A$ through the following expression:
$L_A/L_B = (2.5)^{M_B - M_A}$.
$L_A/L_B = 26/(2.4 \cdot 10^{-3}) \simeq 11 \cdot 10^3$.
We find the magnitude difference by taking the logarithm of the above relation: $log(L_A/L_B) = log((2.5)^{M_B - M_A}) = (M_B - M_A) log(2.5) \simeq 0.4 \cdot (M_B - M_A)$. And $M_B - M_A = log(L_A/L_B)/0.4 = log(11 \cdot 10^3)/0.4 \simeq 10$. Therefore Sirius A and B have a difference in absolute magnitudes of 10.

9.) (a) If one of the stars in the binary system is moving perpendicular to our line of sight, we won't see any radial velocity, because the star is neither moving away nor towards us. It simply stays at the same distance in the plane of its orbit. The radial velocity is zero at all times. For a binary moving in a plane parallel to our line of sight we will see two sinusoidal curves that are symmetric with respect to the x axis, similar to problem 2 in the previous homework set.

(b) This problem has a wealth of results. For simplicity we restrict ourselves to the assumptions that both stars have the same size and the same luminosity, and that they are infinitely far away from us. The two ellipses would then be identical and they have one common focus. The luminosity that we measure would be :
1) L as the stars pass perihelion. The luminosity maintains this value for a short time interval t1, as the stars are both going fast through perihelion.
2) After passage through perihelion, the light curve goes up to 2L and stays at this level until the stars have reached aphelion. This will take a considerably longer interval of time t2.
3) At aphelion the stars are eclipsing each other again and we would measure a drop of luminosity to L. This time the light curves stays on this low level for a time t3, longer than t1 (because the stars are slower at aphelion), but shorter than t2.
4) After passing through the aphelion, the light curve recovers to the maximum value of 2L on which it will stay for an amount of time t2, and the process repeats itself.

If the orbits are arranged such that the semi-major axes are perpendicular to our line of sight, then:

1) The stars will eclipse before passing the perihelion, at one of the points of intersection of the ellipses and the light curve drops down to L and stays on this level for a time t1 (this

time is however longer than that of the previous example, because the orbital speed is larger in this point than at perihelion.

2) The stars will approach perihelion very quickly, as they are moving fastest in this part of their orbits, so the time the light curve stays on 2L after recovering from the eclipse-level L will be very short.

3) This short recovery is followed by a second eclipse when the stars reach the other point of intersection of the orbits. Again the light curve drops to L, and stays there for the same amount of time as during the previous eclipse.

4) Now the stars are moving for a long time on their orbits until they start their cycle again. The light curve will stay at 2L for a time that is longer than any of the intervals measured in the previous example. There is no eclipse at aphelion. In conclusion we would see two brief eclipses that follow one another in a short interval of time, followed by a long period in which the light curve stays on its maximum level.