AY4 Spring 2011: Problem Solving Guide

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This booklet belongs to:

The hardest part of any problem is figuring out how to start, and what the problem is fundamentally about. This guide breaks the process of 'understanding what the question is about' down into steps. You can apply all of the steps listed herein to every problem you'll ever encounter: even those that have nothing at all to do with math, physics, or astronomy.

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The Steps

1. **Read the whole problem.**
2. **Draw a picture.**
3. **Find the key words.**
4. **What units must your answer have?**
5. **Narrow your list of possible equations.**
6. **Start calculating.**
7. **How much sense (if any) does your answer make?**

The Steps, Explained

1. ***Read the whole problem.***

   - If it doesn't make any sense the first time, read it again *out loud*.

2. ***Draw a picture.***

   - It does NOT have to be accurate or complete. It just has to capture what *you think* is going on in the situation that the problem describes.
   - Label everything that you can.
   - If, in the course of working on the problem, you realize that something was wrong with your initial sketch, you can always make another, better sketch.

3. ***Find the key words.***

- In the Key Words section, you'll find a (far from exhaustive or complete) list of words that can clue you in to what the problem is about, and how you should proceed.
- Key Words also include any numerical quantities and/or units that appear in the problem statement.
- Go through the problem, and circle/underline/highlight all of the Key Words you can find.

4. ***What units must your answer have?***

   - Is your answer going to be a temperature, a density, an energy, a ratio, or something else? Figure this out.
   - Write down what *type* of units your answer will have. For example, write down "energy" instead of "ergs," or "distance" instead of "cm".
   - Knowing the units your answer needs to have gives you an idea of what type(s) of equations and formulae are relevant to the problem. For example:
     - units of rate: flux eqns, eqns involving time
     - units of acceleration: acceleration eqns, velocity equations, formulae involving time
     - units of energy: flux eqns, energy formulae
     - units of distance, area, volume, or angle: geometric formulae, orbital/circular motion formulae, wavelength formulae
     - units of number of photons, frequency, or wavelength: energy formulae, radiation formulae

5. ***Narrow your list of possible equations.***

- At this point, you know what units have to appear in your answer. You will also know what units are present in the problem statement. Most importantly, you'll know what units do *NOT* appear in the problem.
- Based on your answer(s) to the previous step, list all of the equations and formulae that could be in any way relevant to the problem.
- Go through that list and eliminate every equation that contains *none* of the units you found in steps 3 and 4.
- Circle all of the 'known' quantities in the remaining equations, and box the variable that represents the thing you want to solve for. If an equation doesn't contain anything that's boxed or circled, it's less likely (but not impossible) that it will be applicable to your problem.

6. ***Start calculating.***

- Look at the equations that remain. Do they have anything in common? For example, are they all geometric relations? Are they all energy equations? Are they all equations that appear in flux problems?
- Which quantities remain constant in this problem? Could any conservation laws (energy, mass, momentum, etc.) applicable?
- Get all the known quantities into the same unit system. For example, convert everything that has units of $M_{\odot}$ to grams. Convert all hours to seconds. Basically, convert everything to cgs units.
- How can your combine these remaining equations to get an answer in the units you want?

7. ***How much sense (if any) does your answer make?***

- Most importantly: does your answer have the right units?
- Convert your answer to more intuitive units, if necessary, to get a gut-level sense of its size. Feet, miles, hours, years, pounds: the English system is okay to use, here.
- Does your answer produce reasonable values for extreme cases? (For example, what happens when one of the variables goes to zero? To infinity?) Does its behavior make physical sense?

Key Words

- energy
- flux
- distance
- speed
- radius
- mass
- time
- ALL UNITS (ergs, miles, cm, decades, molecules, etc.)
- increases/decreases
- factor of/fraction of
- the names of any shapes, and/or their dimensions
- greater/lesser than, more/less than
- eccentricity
- period
- orbit
- focus
- velocity
- acceleration
- force
- luminosity/brightness
- apparent

- angle
- when/where/what
- temperature
- heat
- light
- wavelength/frequency
- “is conserved” or “remains constant”
- “depends on”

(Keep adding your own, here.)

Organizing Your Formulae

We've covered a limited number of topics in this class, which means you only have a certain number of equations and formulae to work with. While that number may not be small, it is finite, and you know that the equations you'll need don't lie outside that set.

Topics covered so far include energy, flux, area, distance, acceleration, and force. Problem techniques that you've had to employ have been limited to geometry, algebra, using conservation laws, trigonometry, and exponents/logarithms.

No matter how gnarly the problem looks, you know you won't have to use calculus, set theory, differential equations, or origami to solve it. You know you already have all of the mathematical tools necessary to solve any problem you encounter in this class. (Real life is almost never so kind, so enjoy this type of certainty while you can.)

During the section where you receive this guide, we will write out and classify all of the formulae and equations that have appeared in lecture so far. As the course continues, you'll be responsible for recording and categorizing every formula that's introduced here in this guide. The next several pages are left blank for this purpose.

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